Assignment 6 - Red in Tooth and Claw

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A biological experiment on Mars has gone astray. Can you predict the eventual outcome? An incredibly useful Python package may help.

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Oops!

TEFLON TUSK, the well-known bazillionaire and loony, has successfully landed a colony ship on Mars. As part of his project to prove that Mars can be made habitable for Terran life, he took with him 100 sheep and 25 hyenas. Before landing, he seeded the planet surface with fast-growing kudzu vines, which quickly covered vast areas (turning Mars from red to green). He then released all the animals to see what would happen.

What happened was that all the animals promptly ran off into the fields of kudzu and have not been seen since. It's up to you to determine what is probably happening to them.

Sheep eat kudzu leaves. Mars now has so much kudzu that it can support any number of sheep. Hyenas eat sheep. The more hyenas there are, the more sheep get eaten. But when the number of sheep decreases, the hyena population decreases also as the hyenas have less to eat, reproduce less often and also die of natural causes. And as the number of hyenas decreases, fewer sheep get eaten and so the population of sheep begins to rise again. That causes the hyenas to reproduce more, so the number of hyenas rises and the cycle starts again.

This type of predator-prey system is described by a pair of equations called the LOTKA-VOLTERRA PREDATOR-PREY EQUATIONS. Each equation gives a "population change" value for one of the two species.

DEFINITION: Let *sheep* be the current number of sheep and *hyenas* be the current number of hyenas. We will use *delta_s* as the change in the number of sheep and *delta_h* as the change in the number of hyenas.

The LOTKA-VOLTERRA equations for *delta_s* and *delta_h* are:

$$delta\_s = k_1 * sheep - k_2 * sheep * hyenas$$
$$delta\_h = k_3 * sheep * hyenas - k_4 * hyenas$$

The four constants in these equations k_1, k_2, k_3 and k_4 are based on the behaviour of sheep and hyenas on Earth.

k_1 is the rate at which the population of sheep increases if left alone.

k_2 is the rate at which hyenas eat sheep.

k_3 is the rate at which the population of hyenas increases.

k_4 is the rate at which hyenas die of natural causes.

Looking at these equations you can see that when the number of hyenas is high, the number of sheep that get eaten will also be high. Similarly, when the number of sheep is high the number of new hyenas born is also high.

These equations are an **abstract** model of predator-prey relationships. In the real world there are other factors that also come into play.

We can track the populations of sheep and hyenas over time like this:

```
sheep = 100
hyenas = 25
list_of_sheep_pops = [sheep]
list_of_hyena_pops = [hyenas]
for i in range(num_years):
        delta_s = k_1*sheep - k_2*sheep*hyenas
        delta_h = k_3*sheep*hyenas - k_4*hyenas
        sheep += delta_s
        hyenas += delta_h
        list_of_sheep_pops.append(sheep)
        list_hyena_pops.append(hyenas)
```

But how can we visualize how the populations are changing over time?

One of the best ways is by using a package called `matplotlib`. This package is extremely powerful and offers a huge number of options. We're going to use it in the most basic way but you will probably want to explore some of its other capabilities for creating both static and interactive graphs, charts and diagrams.

You may need to install it. Here's a page about how to do that: `https://matplotlib.org/stable/users/installing.html`

Here's a very small illustration of how to use matplotlib to draw simple graphs. We define a line on the graph by passing the "plot" function a list of values. It interprets those values as the y-values of a series of points and draws a line passing through all those points. We can add multiple lines to the graph. When all the infomation is added, we use "show" to display the graph.

The x-values are assumed to be [0,1,2,3 ...] but we could override that if we needed to.

```
from matplotlib import pyplot as plt    # it is traditional to use plt as the short name

points_1 = []
points_2 = []
num_points = 500
for i in range(num_points):              # You can make num_points a lot higher than this.
                                         # The graph will scale automatically.
        points_1.append(14*i + 6)
                       # just a random function for illustration purposes
        points_2.append(0.05*i**2 + 21)
                       # another one

fig = plt.figure(figsize=(20,8))
                    # these numbers were chosen to make a big window - you
                    # may find other values to be more suitable for your screen

fig.suptitle("Plotting two Functions")
                    # add a title to the figure

plt.plot(points_1, label="14*i + 6")
                    # add this list of points to the figure, joined by a line

plt.plot(points_2, label="0.05*i**2 + 21")
                    # add the other line to the figure

plt.legend()        # add the labels to the figure
plt.show()          # display the graph
```

This demo program is posted on the Assignment page for your convenience.

The Assignment

Use the Lotka-Volterra equations to model the populations of sheep and hyenas for the next 10000 years.

Scenario 1

According to sheep experts, the values of the four constants are :

$$k_1 = 0.005$$
$$k_2 = 0.0009$$
$$k_3 = 0.0005$$
$$k_4 = 0.02$$

Use matplotlib to create a graph of the results of your modelling process. Save the graph because you need to submit it.

Scenario 2

Hyena experts believe that those four constants are different. They claim these values are more accurate:

$$k_1 = 0.01$$
$$k_2 = 0.0001$$
$$k_3 = 0.0002$$
$$k_4 = 0.03$$

Repeat your modelling with these values.

Question: Which of the two models suggests that Mars will show a fairly stable cycle of rise and fall of the sheep and hyena populations over the next 10000 years?

This is a good example of what we call a chaotic system: small changes in the input can cause large differences in the output.

How You Will Be Graded

The assignment will be marked out of 100. 90% of the grade will be for correctness and 10% of the grade will be for programming style.

The grader will read your code and will run your program to test correctness.

What to Submit

For this assignment, you are required to upload to onQ:

- A Python program containing
    - (a) your implementation of the Lotka-Volterra predator-prey equations
    - (b) your data-gathering for the two sets of values of k_1, k_2, k_3 and k_4
    - (c) your code that uses matplotlib to graph your results.
- A text file or pdf containing your answer to the question stated in the Assignment section
- The two figures (saved as .png files) generated by the given sets of constant values
- You are NOT required to upload the html page generated by pydoc, because I know some students in the class have not been able to get this to work. However, your code must be properly documented with docstrings.

Remember to put your name and student number at the top of your program file, as well as the statement regarding academic integrity (as specified in Assignment 1). Also, your program must contain appropriate docstring documentation at the beginning of the program and in each defined function.

Due Date

The due date for this assignment is 20211114 (November 14), 11:59 PM. This is not the due date originally posted.